

Lemma *Let us consider two functions, say φ and ψ , such that $Q\varphi = \psi$. Then we have*

$$b := \int_E \pi(dx) \psi(x) R_0 \psi(x) \geq 0.$$

PROOF. From the martingale characterization of Markov processes, we have that

$$M_t := \varphi(x(t)) - \int_0^t Q\varphi(x(s)) ds,$$

is a martingale with square characteristic

$$\langle M \rangle_t = \int_0^t [Q\varphi^2(x(s)) - 2\varphi(x(s))Q\varphi(x(s))] ds, \quad t \geq 0.$$