PROBLEMS

1). Let a random flight in R^3 with four directions (generalization of Goldstein-Kac model) has one of the following matrices for one step transition probabilities of swiching Markov process:

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

the case of cyclic change of directions,

$$P = \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

the case of uniform change of directions, and the following evolutionary matrix:

$$A = v \begin{pmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 & 0 \\ 0 & 0 & -\frac{\partial}{\partial x} & 0 \\ 0 & 0 & 0 & -\frac{\partial}{\partial y} \end{pmatrix}$$

Define generator of the limit process when $v = \varepsilon^{-1}$, $\lambda = \varepsilon^{-2}$, $\varepsilon \to 0$.

2). Prove that the Wiener process $w(t)$ is a martingale. Under what conditions it is submartingale/supermartingale?

3). Ensure that in the case of Ornstein-Uhlenbeck process projector on $\mathcal{N}_Q$ and corresponding potential operator satisfy the condition $R_0\Pi = \Pi R_0 = 0$.